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ABSTRACT

A project was conducted at the University of Goteborg in Sweden to study how students of statistics make use of their knowledge when they solve statistical problems and how this knowledge utilization can be improved. The initial study objectives were to identify deficiencies and limitations pertaining to students' knowledge utilization and to formulate principles regarding how students should make use of their knowledge in a better way. The project's purpose was widened to include studies of the nature of the students' statistical knowledge and not just how the knowledge is used. The investigation has provided a foundation for viewpoints on how the transmission of knowledge itself ought to take place and on how the examination can be conducted. A literature survey was conducted which focused on process-oriented research. The literature survey was followed by a more generally oriented report dealing with how process models of problem-solving formulated in psychology can be applied pedagogically to statistical problem-solving, for instance. One area of consideration was the importance of understanding-oriented instruction in problem-solving. Empirical studies were conducted on how students solve statistical problems: attempts to influence the detection of errors by instruction: awareness of errors in problem-solving: and what conditions must exist for problem-solvers to be able to eliminate their errors. Study results are summarized in the form of advice to a problem-solving statistics student. (SW)

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TITLE OF PROJECT: THE PEDAGOGICS OF PROBLEM SOLVING

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BACKGROUND

The goal of an education is usually stated in terms of knowledge and skills. Knowledge is then assumed to mean that information has been transmitted successfully to the students. By skill we often mean something more, namely that the students can make use of the information transmitted, i e their knowledge, in different contexts. The requirement that the students be imbued with skills and not just knowledge is especially important in formal subjects like mathematics and statistics, where a great part of the studies is devoted to problem solving.

Most people have probably met with failure in making use of the knowledge they in some sense possess. A student of statistics may e g know when he or she should use test A and test B, but still use the wrong test in solving a concrete problem. The student may

have neglected to study the problem sufficiently carefully or not have made a sufficient effort to find the right test. Thus it seems likely that there is often room for improved performance as a consequence of better utilization of knowledge and not just as a consequence of acquiring better knowledge.

The general purpose of this project has been to study how students of statistics (a) make use of their knowledge when they solve statistical problems and (b) how this knowledge utilization can be improved. We have seen the project as contributing in this way to the formulation of a pedagogics of problem solving within the subject of statistics. The idea was, first, to study what deficiencies and limitations there are in the students' knowledge utilization and, second, to formulate on the basis of information about these deficiencies a number of principles for how students should make use of their knowledge in a better way.

However, recently the project's purpose has been widened to include studies of the nature of the students' statistical knowledge and not just how the knowledge is used. From the beginning we had intended that the project would primarily result in a number of principles for how statistical problem solving, i e statistical knowledge utilization, ought to take place. As we have now taken an interest in the nature of the knowledge and how this is related to the utilization of the knowledge, we have also obtained a certain foundation for viewpoints on how the transmission of knowledge itself ought to take place and on how the examination can be carried out.

ACTIVITIES PURSUED

STUDIES OF THE LITERATURE

A literature survey (Allwood, 1976) has been written within the project with the intention of elucidating research results concerning factors related to successful problem solving, and proposing on the basis of these results possible ways of improving students' problem solving in statistics.

This report was founded on the notion that it is possible to improve students' problem solving solely by changing their behaviour in the problem solving situation. The report therefore focused on process-oriented research. In such studies it has turned out that the following variables, among others, are important to the result of the solution of mathematical and similar problems: the formulation of the problem when it is given

to the solver; how the solver represents the problem subjectively; the solver's mode of handling abstract concepts; the reading ability, vocabulary, and general linguistic ability of the solver; and the heuristic strategies used. The literature survey also dealt with studies which aimed at trying to improve the result of problem solving by giving the problem solver various kinds of training. Attempts have been made to improve the problem solver's result by, among other things, giving training and instruction in information searching, and in problem solving strategies; linguistic instruction; and by influencing the problem solver to a higher activity level. Studies of training in searching after information in external sources of information show that such training improves the ability to search for information if it is given for a sufficiently long period of time. Training in using efficient problem solving strategies does not have to be given for as long a period to have an effect. It is true of both training for information searching and strategic training that the more explicit information given on what is being trained, the better the results obtained. Another study by Logan & Wodke (1968) concerning training in using problem solving strategies shows that subjects have difficulty avoiding using strategies with a limited area of application in situations where the strategies were not applicable. Studies of linguistic training showed that such training led to improved results in mathematical problem solving. Instruction in an insight problem to search actively for the solution gave rise to improved problem solving compared with a control group. A general result in training studies was that it is possible to improve the problem solver's performance through training. This was particularly true of the type of problems usual in educational situations, i e problems where the problem solver must rely on special knowledge which he or she has learnt previously in the instruction.

On the basis of these research results the literature survey presented proposals of ways to improve students' problem solving ability in statistics. Training in extracting and interpreting information in statistical problems was proposed as being an appropriate way of improving the results of problem solving in the initial stages of problem solving. This training ought both to be purely linguistic-semantic and to concern information strategies (e g not to represent the facts in the text sequentially in the order they are in but to try instead to produce an integrated interpretation). In connection with this it can be pointed out that subsequent studies of algebraic word problems have indicated the importance of giving students training in recognizing different types of problem (Hinsley, Hayes, & Simon, 1977). The results show that if the problem solver succeeds in identifying the type of problem he or she tends to make a more integrated interpretation of the text compared to problem solvers who do not succeed in

identifying the problem type. These latter tend instead to interpret the problem instructions sequentially line by line. Moreover, Hayes, Waterman, and Robinson (1977) have shown that subjects' assessment of what information is important in the problem text is determined to a great extent by what type of problem they think they are solving.

Allwood (1976) also took up the proposal of improving students' problem solving by teaching good problem solvers' ways of solving problems to bad problem solvers. Schoenfeld (1979) has described a course in problem solving completely modelled on teaching how experts solve the problems.

Allwood (1976) discussed finally the question of the desirability of individualising training in problem solving. Several studies described in the report showed that problem solvers have different cognitive styles in their problem solving. Furthermore, one conclusion from a study by Osler & Weiss (1962) is that intelligent problem solvers would not gain so much from purely linguistic training as a means of improving their problem solving. Allwood (1976) proposes the possibility of diagnostic instruments by means of which individual problem solvers' weaknesses are diagnosed, after which adequate training can be given. In a similar way Davis & Cooney (1979) have suggested that students' errors in problem solving be classified and an error profile drawn up for each student, whereupon the student can be given such training as his errors show he needs.

The literature survey was followed up by a more generally oriented report (Allwood & Montgomery, 1976 /b/), which deals with how process models of problem solving formulated in psychology can be applied pedagogically to e g statistical problem solving. The authors note to begin with that psychological process models have not taken account of the question of motivation. Despite this it can be said that to the extent they have been successful in improving problem solving they in all likelihood influence students' problem solving motivation to the better.

Among other things, the report discussed the importance of understanding-oriented instruction in problem solving. An advantage of such instruction is that the resultant ability to solve problems lasts longer than would otherwise be the case. Moreover, it has been shown that understanding-oriented instruction leads to the problem solving ability becoming more general in the sense that the problem solver gains a greater ability to solve problems differing in various respects from the problems he or she has previously learnt to solve. Furthermore, it can be added that a greater understanding of

problems ought to give the problem solver more possibilities of checking that the solution to the problem is correct and greater possibilities of remembering the next step in the solution of the problem.

In the report a number of different general problem solving strategies (comprehensive ways of solving problems) were also discussed. One of the strategies which was discussed was the means-goals strategy, which means that the problem solver is not to carry out any step in problem solving without having a clear purpose for doing so. In connection with the description of different strategies the absence of explicit presentations of strategies for problem solving in e g mathematics books for the comprehensive and the secondary school was noted with astonishment.

Finally a general problem solving model based on the means-goals strategy - "the general problem solver" (GPS) presented by Newell & Simon (1972) - was described. The authors suggested that GPS could be used in several ways in applied contexts. Presentations of problems in textbooks and instruction could be structured according to GPS's principles. Moreover it ought to be possible to teach GPS's way of solving problems to students as a general problem solving strategy and to enable them to find suitable ways of seeking information in their memories.

EMPIRICAL STUDIES

How do students solve statistical problems?

With the purpose of gaining a clearer picture of statistics students' problem solving behaviour Montgomery and Allwood (1978 /a/) collected think-aloud data from 19 statistics students who solved individually three exam-type statistics problems. After the problem solving the students had to answer some knowledge questions concerning facts relevant to the problems. The think-aloud method involves the problem solver being given instructions to verbalize all his thoughts while solving the problem. These verbalizations are tape-recorded and transcribed; the transcription is subsequently analyzed. This method of data collection has shown itself to be able to give an enormously detailed picture of the problem solver's mental processes during the problem solving process, at the same time as it hardly disturbs the problem solver or distorts the process at all (Ericsson & Simon, 1979). As the authors were most interested in bad problem solvers' problem solving processes, but even in a comparison between good and bad problem solvers, the students were chosen so that 13 students had low grades in mathematics and little mathematical experience and 6 students had high grades and considerable experience of mathematics.

The problems used were two demographic problems and a price trend problem. It turned out that the problem solvers developed one or more attempts at solutions of the problems. These attempts were classified in three categories with respect to their degree of correctness and quality. The results show that in the cases the problem solvers developed several problem solving attempts, later attempts tended to be better than earlier. This result shows that the problem solver's ability to discover his errors is important to success in problem solving. Slightly less than half the problem solvers who finally gave a correct solution had initially an incorrect attempt at solving the problem. This result also showed that the students had the ability to generate information during the problem solving process which they did not have access to in the initial stages.

The results of the two demographic problems showed that problem solvers with correct problem solving attempts more often gave more complete definitions of a central concept in the problem's interrogative sentence. Moreover, in one of the demographic problems the good problem solvers referred more often to what was asked for in the problem than the bad problem solvers did. The third problem, the price trend problem, was formulated in such a way that it could be misinterpreted if read carelessly. The result here showed that the students who interpreted the problem text incorrectly were, at the same time, the students with the poorest knowledge of the problem area according to the answers to the knowledge questions the students answered after solving the problem. This result could mean that better knowledge of the area a problem text deals with facilitates correct understanding of the text, and thereby also increases the student's ability to solve the problem.

To sum up, the results of this study can be said to point to the importance of the students increasing their activity during problem solving as regards the understanding of the problem text and concerning their awareness of how the solution produced is related to what was asked for in the problem.

Attempts to influence the detection of errors by instruction

The next study in the project - Allwood, Montgomery (1977) - aimed at testing the possibility of activating problem solvers during problem solving with the help of written instructions on problem solving. As the previous study had shown that the ability to find and correct errors made during problem solving is important to successful

problem solving, this study focused on the students' handling of errors made previously. Ten students of statistics were instructed to think aloud while they solved two statistical problems. Students with a poor or average knowledge of mathematics were selected to take part in the study. Each student was also given a set of problem solving instructions which gave general directions as to how he or she should act in different stages of problem solving. The study focused on analyzing the effect of three instructions which aimed at getting the problem solver to check his or her solution and which, according to the instructions, were to be used after each step in solving the problem. The use of these instructions was only analyzed after the steps where the students made an error in their solutions. The concept "step" was defined intuitively for the students at the beginning of the experiment, and the students were reminded if they neglected to use the instructions at the right place in the experiment.

The results show that the students made 55 errors during the problem solving process; 25 of these were detected in the sense that the errors were replaced by a new alternative solution. 20 of the 25 errors detected were replaced by a correct new solution. 13 of the 25 were detected when the students followed the instructions for problem solving, 9 of them when the students carried out the instructions for checking. An analysis of think-aloud data in connection with the detection of the errors detected showed that 10 errors were noticed because the solution gave rise to such extreme values in the result that the students reacted. 6 errors were detected by so-called spontaneous detection, which means that the students, immediately after making the error, changed the solution to another alternative solution without more explanation than that the solution was wrong. 5 errors were detected because the problem solver was displeased with other aspects of the answer than its numerical value. 4 errors, finally, were detected when the student, to be on the safe side, repeated his or her solution of the problem. It is worth noticing that all the detections of errors of the last-mentioned kind took place when the students followed the instructions for checking.

The content of the students' replies to the instructions for checking were analyzed with regard to to what extent they succeeded in getting the students to activate their knowledge when they followed the instructions. Here two aspects were taken into consideration: on the one hand, the degree of specificity of the knowledge activated and, on the other hand, what type of knowledge the students activated.

In general it can be said that the students' responses to these instructions were marked by a low degree of activity or cognitive effort. For instance, the students made no assessment of their solutions at all in

a third of the responses analyzed. The number of omissions became increasingly large for later instructions. Only a third of the responses were coded in the category high specificity, which was defined as responses more specific than a general assessment of a certain well-defined part of the solution. Even the analysis of what type of knowledge the students activated (done on two of the three instructions) showed that the students avoided cognitive effort. For instance, when the students were given an open-ended instruction "check that your solution has been carried out correctly", they tended, when they did more than make a general assessment, to concentrate on their numerical calculations in more than half the cases. This activity can be supposed to demand little cognitive effort as the arithmetical knowledge can be regarded as well-learned and not demanding any great amount of reflection when used. A more specific instruction "look for alternative solutions" turned out to be more successful in getting the students to consider other aspects of their solutions than purely numerical ones like e g considering different grounds for choosing a particular formula.

One conclusion the study gave rise to was that the instructions for checking had had some effect on the students' tendency to detect errors, but that the students' tendency to minimize their cognitive effort means that instructions for problem solving should contain guidance as to how the activities the instructions request should be carried out.

Awareness of errors in problem solving

The next study in the project (Montgomery & Allwood, 1978 /b/) aimed at elucidating in greater detail the question of the reasons for the students often not eliminating their errors. The study started from the notion that this could depend either on the problem solver trying but failing to eliminate the error or on the problem solver not even noticing the error. By having the problem solvers assess the degree of correctness of the different steps in their solutions, the study wanted to examine how aware inexperienced problem solvers in statistics are of what parts of their solutions are correct and what parts incorrect. In more detail, the experiment was arranged so that 25 students first had to solve two statistical problems; then they had to assess the correctness of the successive parts of their solution of the problem on a five-point scale. The students were also asked to try to specify the error and suggest a new alternative solution to the parts of the solution they did not assess

as "completely sure that I have solved the whole of this part correctly".

Seen generally, the results showed that the students had a poor ability to recognize correctly and incorrectly solved stages. More than half of the incorrectly solved stages were assessed by the students as entirely correct "with complete certainty". The results showed, in addition, that stages in the solution with numerical computational errors and serious formula errors were more often suspected to be incorrectly solved than stages with slight formula errors. There was no clear difference between good and bad problem solvers in their ability to assess the correctness of parts of their solutions. An analysis was made of the students' attempts to analyze what was wrong with the stages of the solution they had assessed as incorrect. Here it turned out that the students had too strong a tendency to assume they had made computational errors and too weak a tendency to assume they had made formula errors, compared with the number of errors of the two kinds they actually had made.

The authors speculated about possible sources of error which can lie behind the students' misjudgements. One likely source of misjudgements of the quality of the solution is that the problem solvers have quite simply omitted to check parts of their solution. The results from the study of Montgomery and Allwood (1977) described previously supports this hypothesis. Another source of error may be that the students have indeed checked a step in the solution but recognized it as correct when it is in fact incorrect. This may, for instance, depend on the students not being sufficiently aware of the reconstructive nature of memory. Finally the students' incorrect assessments may have depended on their finding it easier to imagine that they had made certain types of errors than other types of errors, e g on account of the tendency people exhibit to assess the probability of an event on the basis of how easy it is to imagine events of similar kinds (Tversky & Kahneman, 1973).

Knowledge, technique, and the detection of errors

The purpose of the last study carried out in the project (Allwood & Montgomery, 1978 /a/) was to elucidate why problem solvers make errors in their solutions and to study more closely what conditions must exist for problem solvers to be able to eliminate their errors. The study's purpose was to try to investigate to what extent the problem solver's deficiencies in technique and in knowledge cause difficulty in the above-mentioned respects. Here it was assumed that the smaller cognitive deficiencies are and the more important deficiencies in technique are in the explanation of the origin of the error, the greater should be the problem solver's chances of correcting his or her errors. The

technique of problem solving can be characterized as the way in which the problem solver makes use of the different pieces of knowledge he possesses in order to solve the problem. Here such aspects as carefulness come in, as do aspects like completeness in searching for different possible solutions, how knowledge is sought in the memory, and how the solution is evaluated - important aspects of the problem solver's problem solving technique.

The problem solver's knowledge was assumed to be of two kinds: propositional and algorithmic (Greeno, 1973). By propositional knowledge we mean knowledge about events, and relations between different concepts and events. Algorithmic knowledge was assumed to be of two kinds: on the one hand, procedural knowledge (i e knowledge about different ways of carrying out the steps in problem solving) and, on the other hand, knowledge about the conditions for application of the different ways of carrying out a step in the solution.

The study was arranged so that 20 students first had to solve two statistical problems, whereupon they were given the task of explaining their solutions with justifications of each step to an imaginary listener with only upper secondary school mathematics. The instruction for this part of the experiment also asked the student to say if he detected an error in his own solution. Next the problem solver was questioned closely about his reasons for his solution of each step in the solution. Gradually the questions were focused especially on the incorrectly solved parts of the solution. Each student went through the experiment individually. The idea of having the students recount their solutions was to investigate whether this counteracted the problem solvers' tendency to minimize their cognitive efforts in checking a solution to a problem.

The results showed that the technique of having the problem solvers explain and justify their solutions only had a marginal effect on their detection of errors. Only three errors, 4 % of the 80 errors made, were detected in this way. Most steps in the solution were not evaluated at all with regard to correctness by the students. However, the students gave a negative evaluation of incorrectly solved steps (27 %) rather more often than of correctly solved steps (8 %).

An analysis was performed to see what type of information the students conveyed when they explained their solutions. Here it turned out

that to quite a large extent (62 % of all steps) the students gave a complete description of how they had solved the step and that they gave reasons for their solution of the step to a considerably smaller extent. 43 % of the incorrect reasons would have been correct if the problem solver had solved the step correctly. This would suggest the students have a tendency to reproduce mechanically explanations they have learned for a step in a solution without understanding properly the relation between the solution of the step and the explanation given. Correlation calculations showed that good problem solvers gave complete, correct reasons for their solutions and a full description of their solution more often than bad problem solvers. To sum up, the "narrative technique" turned out not to be able to break the students' tendency to minimize their cognitive efforts.

In the part of the experiment where the students were questioned by the experimenter about the reasons for their solutions 27 errors were detected and 20 of these eliminated. Nine of these were eliminated after only one to three questions from the experimenter. Analysis of the questions showed that in these nine cases their function was solely to draw the student's attention to a particular part of the solution. For the remaining 11 errors eliminated the average number of questions was 21. In none of the above-mentioned cases did the experimenter give concrete information about statistical facts in his questions.

In the study an attempt was also made to assess to what extent deficiencies in the problem solvers' knowledge and technique components contributed to the errors they made in their problem solving. It turned out that at least a third of the errors made could have been avoided if the students had had a more efficient problem solving technique. Half these errors were the kind of error the students must reasonably be considered to have sufficient knowledge to avoid. This applies above all to arithmetical errors and errors in transcription. The other half of the errors which can primarily be regarded as having arisen on account of deficiencies in the students' problem solving technique is the 15 non-elementary errors the students eliminated when they were questioned by the experimenter.

Yet another important type of technical defect in the students' problem solving emerged from the analysis of the data. On several occasions the students showed by their answers to the experimenter's questions that they solved the problems "the way they remembered" from previous problem solving or instruction that they should be solved. That this way of generating new steps in the solution of a problem is defective was shown by the fact that the students explained 8 of their incorrect solutions in this way. This type of problem solving technique makes it more difficult for the problem

solvers to detect their errors as their attempts to explain their solutions will come to an end when they imagine they remember things from previous instruction or problem solving.

Regarding the two thirds of the students' problem solving errors in the experiment which were related to deficiencies in knowledge, an analysis of the students' problem solving errors was carried out to illuminate more exactly the kind of deficiencies in knowledge concerned. The analysis showed that more than half the deficiencies in knowledge the students displayed in connection with their errors were deficiencies in propositional knowledge. The major part of the deficiencies in propositional knowledge turned out to be in the form of incorrect conceptions rather than in the lack of any conception at all. As to the students' deficiencies in procedural knowledge, the reverse was true. That is, there were, here, considerably more cases of the lack of any conception at all than cases of incorrect conceptions. It was considered that these facts indicated that the students adopted a different attitude to procedural knowledge than to propositional knowledge. Together with other findings in the study like e g the students' resistance in many cases to changing their mode of solution (procedural knowledge) these data were considered to indicate that the students had less respect for knowledge of the propositional type than for procedural knowledge. The pedagogical implications which follow from this are that inexperienced problem solvers in statistics should be able to become more successful if they devote more time to considering alternative modes of solution and to evaluating critically the implications of their propositional knowledge.

Uninformative or identical designations of different concepts stand out as one reason for the appearance of incorrect propositional knowledge. Another reason seems to have been that the students confused concepts with similar meanings. Here it was presumed that more informative and more dissimilar names for important concepts and greater emphasis in the instruction on differences in the meaning of concepts similar in meaning might increase problem solving ability.

To sum up, the results of the study pointed to the importance of more understanding-oriented instruction and problem solving for the achievement of good results in problem solving.

CONCLUSIONS

The solution of statistical problems depends to a great degree on knowledge. You could say that in the project we have studied fragile knowledge and the way it is used in problem solving (Montgomery, 1978). A key concept in this context is understanding. The more understanding, the more coherent the knowledge is, the less fragile the knowledge is and the greater the possibilities of using the knowledge are. The project has also focused on another concept, which is, however, closely related to understanding, viz the detection of errors. Thus the good problem solver solves problems in a more understanding-oriented way than the bad problem solver and he or she is better at detecting and correcting his or her errors. Moreover, our results show clearly how deficient understanding is a handicap where the possibility of detecting errors in problem solving is concerned.

Our results also indicate that errors in problem solutions originate not only in deficient understanding but also in a lack of will, ambition to figure out - try to understand - how to solve a part of a problem or check the correctness of a solution. You could say that our experimental subjects have been cognitively lazy, i e they have had a tendency to minimize their intellectual effort. If an intellectual task is felt to be stimulating, the opposite may be more likely to be the case. Here we are getting on to what we believe to be an important problem: How are we to make statistics - and understanding of statistics - something interesting?

Our results could be summarized in the form of the following advice to a problem solving statistics student:

1 Think through the presentation of the problem carefully. Define central concepts mentioned in the problem.

2 When you solve the problem check continuously that your solution follows what is asked for in the problem.

3 Be sceptical about whether you have solved the problem correctly. You probably have more (serious) errors than you believe spontaneously. Above all suspect that you have used an incorrect method (rather than that you have made an arithmetical error).

4 Check that you have understood each step in the solution. Do this actively by producing the knowledge which shows that you have understood the step in question.

5 If you have difficulty understanding what you have done, suspect you have made an error. Do not rely on your recollection of what a solution should look

like if you do not understand at the same time.

6 If you suspect an error or if you have got stuck, do not give up. You often have considerable possibilities of figuring out the correct solution.

In most cases it is probably insufficient only to give directions of the above type to the students. In actual fact several of our studies show that simple instructions or directions have limited effects on the experimental subjects' thinking. The effects appear to be limited by, among other things, the students' tendency towards cognitive laziness. One way of counteracting this problem is to give directions so specific that it is difficult to justify that one is following the directions by carrying out activities requiring little intellectual effort. The fact that more specific directions can pay is also supported by the results from one of our studies. Training in using the directions is doubtless also important.

If we want the students to be oriented towards understanding, the instruction should of course also be oriented towards understanding. To say that it is important that instruction imparts understanding may appear to be to state a truism. On the other hand, opinions can differ about what kind of understanding the instruction is to convey. What we believe is important is that the students understand every step in different statistical methods. One way of increasing this understanding, which was touched on in one of our studies, is by suitable designations for the different steps in a method. Consequently designations should be as informative as possible and should not invite confusion.

Thus the relation between theory and practice should be understood and understood not only at a general level but also with regard to every step in the method. In that way the students are given a greater possibility of remembering and generalizing their methodological knowledge - which is an old truth - but also increased possibilities of detecting errors - which this project in particular has demonstrated.

The examination ought also, in our opinion, to focus on the understanding of the relation between theory and method. Thus we can have questions about why a particular method looks like it does. By "why" we do not only mean mathematical derivations. For even these can be learned without being understood. Thus it is a question

of framing the demands on understanding in such a way that we teach and examine real understanding. In that way statistics can perhaps become more interesting too.

Carl-Martin Allwood and Henry Montgomery

1979-06-27

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